2. Measurement theory

Temperatures above the freezing point of silver (1234.93 K or 961.78 °C) are defined on the International Temperature Scale of 1990 (ITS-90) [2] in terms of the ratio of spectral radiances of two blackbody sources, one of which is maintained at the temperature of freezing silver, gold (1337.33 K or 1064.18 °C), or copper (1357.77 K or 1084.62 °C). The 1990 NIST Scale of Radiance Temperature (1990 NIST) is a realization of the ITS-90 using a gold fixed-point blackbody. In this section, the blackbody temperature will be defined in terms of the spectral radiance. Using the signal measurement equation, the measurement equation for the calibration of a transfer standard will be derived.

The signal measurement equation, defined by Nicodemus and Kostkowski in 1978 [5], relates the detector signal output, S [V][2], to the source flux input parameters through a detector responsivity term, R_l [A·W^{-1}], by the integral relationship,

$$S = \int_A \int_w \int_l R_l \cdot L_l \cdot \mathrm{d}\,l \cdot \cos q \cdot \mathrm{d}w \cdot \mathrm{d}A \,, \tag{1}$$

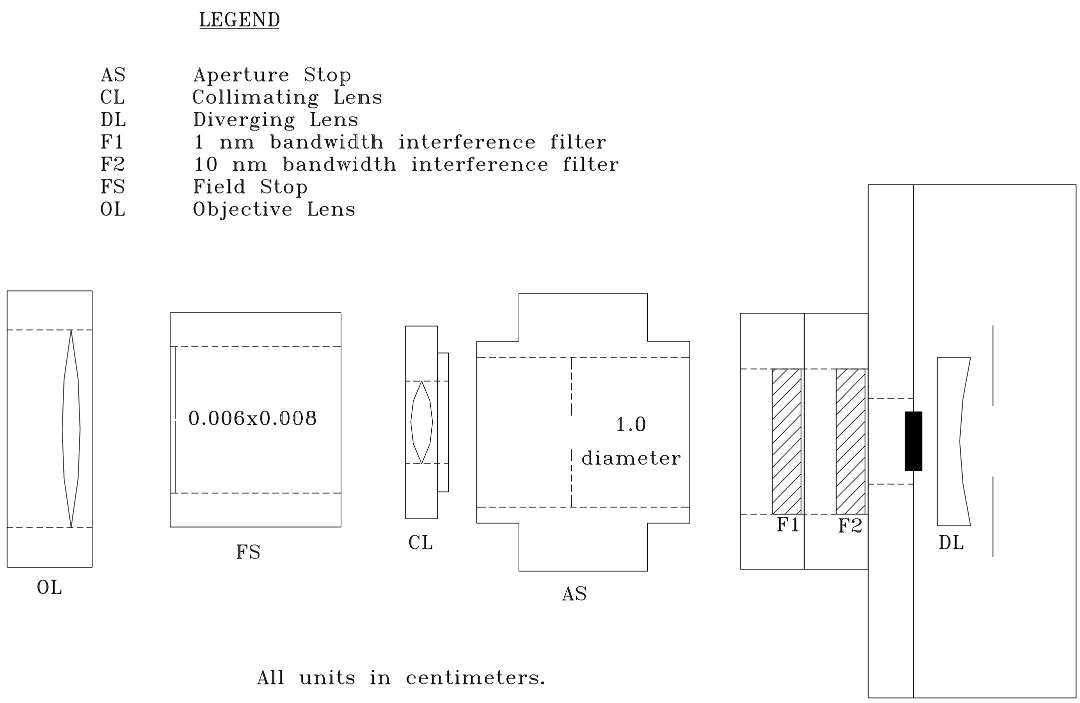

Figure 1. Schematic of optics for the NIST photoelectric pyrometer.

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[2] As an aid to the reader, the appropriate coherent SI unit in which a quantity should be expressed is indicated in brackets when the quantity is first introduced.

where L_l [W·m^{-3}·sr^{-1}] is the spectral radiance, d l [m] is the wavelength band, q [rad] is the angle between the aperture normal and the line connecting the aperture centers, dw [sr] is the differential solid angle originating at the source aperture as defined by the detector aperture, and dA [m^2] is the differential source aperture area. For the NIST PEP in figure 1, the spectral responsivity R_l includes the spectral transmittance of the interference filters, the spectral transmittance of all other optical elements, and the spectral responsivity of the detector. In terms of its specific components, the spectral responsivity is

$$R_l = t_{l,\mathrm{OL}} \cdot t_{l,\mathrm{CL}} \cdot t_{l,\mathrm{F1}} \cdot t_{l,\mathrm{F2}} \cdot t_{l,\mathrm{DL}} \cdot t_{l,\mathrm{EC}} \cdot R_{l,\mathrm{PEP}} \, , \tag{2}$$

where $t_{l,\mathrm{OL}}$ is the spectral transmittance of the objective lens, $t_{l,\mathrm{CL}}$ is the spectral transmittance of the collimating lens, $t_{l,\mathrm{F1}}$ is the spectral transmittance of the 1 nm bandpass interference filter, $t_{l,\mathrm{F2}}$ is the spectral transmittance of the 10 nm bandpass interference filter, $t_{l,\mathrm{DL}}$ is the spectral transmittance of the diverging lens, $t_{l,\mathrm{EC}}$ is the spectral transmittance of the evacuated window cell, and $R_{\lambda,\mathrm{PEP}}$ is the detector absolute spectral responsivity.

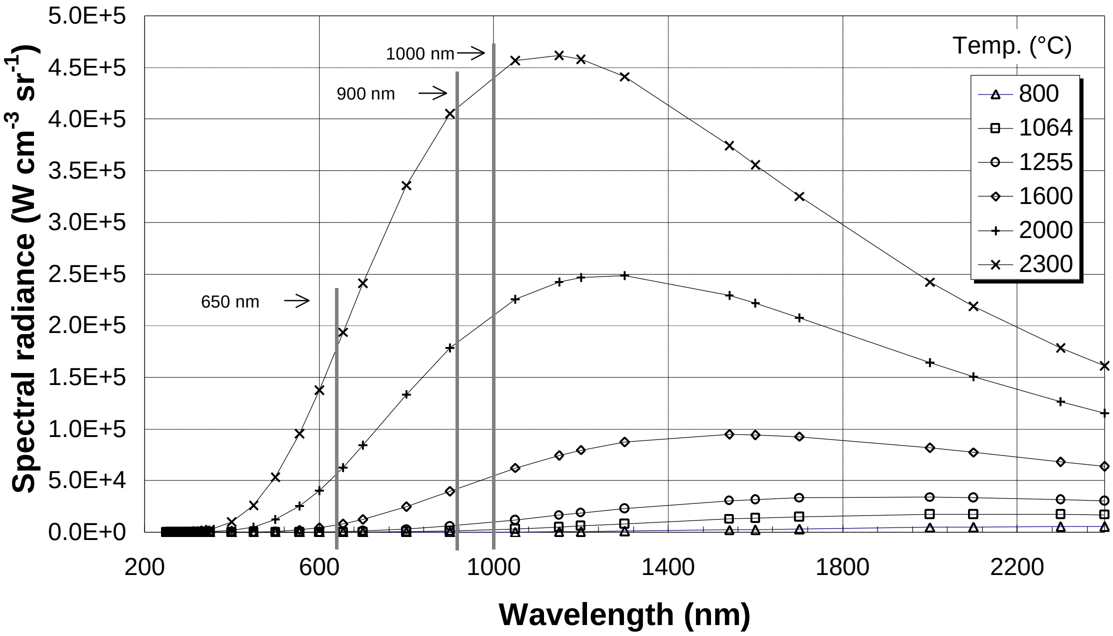

Figure 2. Blackbody spectral distribution. As the temperature increases, the peak moves towards shorter wavelengths, and the slope increases at each wavelength.

Spectral radiance L_l is the radiant power contained in a defined area, solid angle, direction, and wavelength interval,

$$L_l = d^3\Phi / dA \cdot \cos a \cdot db \cdot dl \ , \tag{3}$$

where F is the radiant flux [W], A is the source area, a is the plane angle between the surface normal and the direction of propagation [rad], b is the solid angle about that direction [sr], and l is the wavelength [m]. A few blackbody distributions from 250 nm to 2500 nm between 800 °C and 2300 °C are illustrated in figure 2. For an ideal blackbody in a vacuum environment, the relation between spectral radiance, wavelength, and temperature is given by the Planck equation [6],

$$L_l = \frac{c_{1L}}{l^5 \cdot \left(\exp\left(c_2 / \left(l \cdot T\right)\right) - 1\right)} \ , \tag{4}$$

where c_{1L} is the first radiation constant in radiance form with a value of 119.1044 W·nm^2, c_2 is the second radiation constant with a value of 1.4388×10^{-7} nm·K [7], l is the wavelength in vacuum, and T is the temperature [K]. The Planck equation in the form of eq (4) is the definition for an ideal blackbody radiator. When using a non-ideal blackbody such as a fixed-point blackbody to realize the temperature scale, the following form of the Planck equation is used,

$$L_l = \frac{e_l \cdot c_{1L}}{n_l^2 \cdot l^5 \cdot \left(\exp\left(c_2 / \left(n_l \cdot l \cdot T\right)\right) - 1\right)} \ , \tag{5}$$

and temperature is defined as a function of spectral radiance using the following equation

$$T(l, L_l) = \frac{c_2}{n_l \cdot l \cdot \ln\left(1 + e_l \cdot c_{1L} / n_l^2 \cdot l^5 \cdot L_l\right)} \ , \tag{6}$$

where e_λ is the spectral emissivity of the blackbody (which is equal to unity in the case of an ideal blackbody), 8 is the wavelength in air, and n_l is the refractive index of air at 15 °C and 101,325 Pa. From the Cauchy [8] formula,

$$n_l = 1 + \left(2726.43 + 12.288\ \mathrm{nm}^2 / \left(l^2 \times 10^{-6}\right) + 0.3555\ \mathrm{nm}^4 / \left(l^4 \times 10^{-12}\right)\right) \times 10^{-7} \ , \tag{7}$$

the refractive index of air at 655.3 nm is about 1.00028. The temperatures described in this document are radiance temperatures. The radiance temperature of a radiator is equivalent to the temperature of a blackbody with the same radiant intensity of the radiator's surface at a specified wavelength. The relationship between the radiance temperature and the true or thermodynamic temperature of a blackbody is given by:

$$\frac{1}{T} = \frac{1}{T_l} + \frac{l}{c_2} \cdot \ln e_l \ , \tag{8}$$

where T is the thermodynamic temperature, T_8 is the radiance temperature, *8* is the mean effective wavelength of the NIST PEP (655.3 nm), and e_λ is the estimated emissivity of the blackbody (0.99).

Determination of the spectral radiance temperature of a working standard (WS) lamp requires measurement of the ratio r_1,

$$r_1 = \frac{S_{\mathrm{WS}}}{S_{\mathrm{Au}}} \ , \tag{9}$$

of the signals from the transfer standard and the goldpoint blackbody (Au) with the NIST PEP. From eq (2), this measured ratio is actually

$$r_1 = \frac{\int\limits_{l}\int\limits_{A_s}\int\limits_{w} R_l \cdot \left(L_l \cdot \cos q_{\mathrm{s}} \cdot \mathrm{d}w \cdot \mathrm{d}A_{\mathrm{s}}\right)_{\mathrm{WS}} \cdot \mathrm{d}\,l}{\int\limits_{l}\int\limits_{A_s}\int\limits_{w} R_l \cdot \left(L_l \cdot \cos q_{\mathrm{s}} \cdot \mathrm{d}w \cdot \mathrm{d}A_{\mathrm{s}}\right)_{\mathrm{Au}} \cdot \mathrm{d}\,l} \ . \tag{10}$$

To simplify the complex expression in eq (10), it is assumed that the spectral radiances $L_{l,\mathrm{WS}}$ and $L_{l,\mathrm{Au}}$, spectral responsivities $R_{l,\mathrm{WS}}$ and $R_{l,\mathrm{Au}}$, and amplifier gains G_{WS} and G_{Au} are independent of both direction and spatial location. Furthermore, these three variables can be defined by unique for the transfer source and the gold-point blackbody at equivalent wavelengths (to be defined later) over the same small wavelength band d *l*. The solid angle terms can be replaced with the definition of the solid angle,

$$w = \frac{A_d \cdot \cos q_d}{D^2} \ , \tag{11}$$

where A_{d} is the detector area [m^2], q_{d} is the angle between the optical axis and the normal to the detector surface [rad], and D is the distance between the detector area and the source area [m]. Assuming that the areas are independent of direction and that the solid angles are independent of area or spatial location, this ratio then becomes

$$r_1 = \frac{\left(R_l \cdot L_l \cdot A_{\mathrm{s}} \cdot \cos q_{\mathrm{s}} \cdot A_{\mathrm{d}} \cdot \cos q_{\mathrm{d}}\right)_{\mathrm{WS}}}{\left(R_l \cdot L_l \cdot A_{\mathrm{s}} \cdot \cos q_{\mathrm{s}} \cdot A_{\mathrm{d}} \cdot \cos q_{\mathrm{d}}\right)_{\mathrm{Au}}} \cdot \frac{D_{\mathrm{Au}}^2}{D_{\mathrm{WS}}^2} \ , \tag{12}$$

where A_{s} is the source area [m^2], q_{s} is the angle between the optical axis and the normal to the source surface [rad], $R_{l,\mathrm{WS}}$ is the detector responsivity when viewing the transfer source, and $R_{l,\mathrm{Au}}$ is the detector responsivity when viewing the gold-point blackbody. The spectral responsivities $R_{l,\mathrm{WS}}$ and $R_{l,\mathrm{Au}}$ are assumed to be equal. If the source aperture areas dA_{TS} and

dA_{Au}, source solid angles $d\omega_{WS}$ and $d\omega_{Au}$, and the source inclination angles θ_{WS} and θ_{Au} are the same for measuring both the transfer standard and the gold-point blackbody, the measured ratio simplifies to the expression,

$$r_1 = \frac{L_{\lambda,\mathrm{WS}}}{L_{\lambda,\mathrm{Au}}} = \frac{L_\lambda(T_{\mathrm{WS}})}{L_\lambda(T_{\mathrm{Au}})} \,. \tag{13}$$

This relationship is the defining equation for the ITS-90 above 1337.33 K. In terms of eq (4), it can be written at a discrete wavelength, λ, as

$$r_1 = \frac{L_\lambda(T_{\mathrm{WS}})}{L_\lambda(T_{\mathrm{Au}})} = \frac{\exp\left(c_2 \,/\left(n_\lambda \cdot \lambda \cdot T_{\mathrm{Au}}\right)\right) - 1}{\exp\left(c_2 \,/\left(n_\lambda \cdot \lambda \cdot T_{\mathrm{WS}}\right)\right) - 1} \,, \tag{14}$$

where $L_\lambda(T_{WS})$ and $L_\lambda(T_{Au})$ are the spectral radiances of the two blackbodies at temperatures T and T_{Au}, T_{Au} is the temperature of freezing gold defined as 1337.33 K, and r_1 is their ratio. In principle, a measurement of the ratio at a discrete wavelength with a linear response instrument yields the value of T.

The radiance temperature scale is typically maintained and disseminated on tungsten ribbon filament lamps, which possess a repeatable lamp current versus radiance temperature relationship. At the NIST, a pyrometer system is presently being used with a mean effective wavelength of 655.3 nm. This method requires that the pyrometer relative spectral response extends only over a small spectral range, or is known accurately enough to determine the wavelength at which the integrands of eq (10) have the same ratio as the integrals. Equation (13) above is an approximation, and, in practice, corrections, which will be presented in the next section, are used.